



## DYNAMICAL SYSTEMS: THEORY AND NUMERICAL METHODS

Enrollment year	2017/2018
Academic year	2017/2018
Regulations	DM270
Academic discipline	MAT/08 (NUMERICAL ANALYSIS)
Department	DEPARTMENT OF ELECTRICAL, COMPUTER AND BIOMEDICAL ENGINEERING
Course	BIOENGINEERING
Curriculum	Tecnologie per la salute
Year of study	1°
Period	1st semester (02/10/2017 - 19/01/2018)
ECTS	6
Lesson hours	56 lesson hours
Language	Italian
Activity type	WRITTEN AND ORAL TEST
Teacher	PAVARINO LUCA FRANCO (titolare) - 6 ECTS
Prerequisites	Basic mathematical courses of the "laurea triennale" or "undergraduate degree" and or "bachelor degree"
Learning outcomes	<p>The aim of the course is divided in two parts. DYNAMICAL SYSTEMS: theory and numerical methods (6CFU) and FINITE ELEMENT METHOD AND APPLICATIONS (3CFU).</p> <p>The first part of the course introduces the main concepts related to qualitative and quantitative study of solutions of ordinary differential systems providing the main analytical and numerical methods for the investigation of the dynamics of mathematical models and the critical interpretation of the numerical results.</p>
Course contents	<p>DYNAMICAL SYSTEMS: theory and numerical methods.</p> <p>The course is an introduction to the solvability of initial value problem for</p>

ordinary differential systems and to the investigation of the qualitative properties of solutions and of equilibrium points with their asymptotic behaviour. The course develops the numerical methods for the numerical simulation of dynamical systems with applications to population dynamics and bistable models.

#### FINITE ELEMENTS METHODS AND APPLICATIONS

The course introduces the basic notions of the Finite Element Method and its theoretical grounds. Moreover, the practical part of the course will be devoted to the implementation of a MATLAB solver for elliptic problems in two dimensions.

#### DYNAMICAL SYSTEMS: theory and numerical methods

##### Basic notion of linear algebra and analysis

Vectorial spaces, matrices, eigenvalues, eigenvectors, linear differential equations, differential and integral calculus, vectorial Taylor development.

Introduction to initial value problems for ordinary differential equations  
Local and global solvability, continuous dependence on the initial data, parameters and right hand side perturbations

##### Asymptotic Stability

Stability of solutions and of equilibrium points. Linear systems. Stability of the linear autonomous systems based on the spectral abscissa.  
Nonlinear system: linearization. Nonlinear system: Liapunov function.  
Two dimension linear system and global analysis of the phase plane.

##### Basic notions of numerical analysis

Polynomial interpolation and remainder terms. Numerical integration: Newton-Cotes formulae and Gaussian quadrature. Functional iteration for a system of nonlinear equations: explicit iteration scheme and Newton method.

##### Numerical methods for ordinary differential systems

One step methods: consistency, zero-stability and convergence.  
Runge-Kutta methods based on numerical quadratures, Runge-Kutta methods based on collocation methods. Linear multistep methods: consistency, zero-stability and convergence. Adams Bashforth and Moulton methods, Predictor-Corrector methods, backwards differentiation formulae. Estimators of the local discretization error and adaptive strategy of the time step. Test problems and region of absolute stability. Stiff problems.

Introduction to bifurcation involving fixed points and limit cycles in biological systems.

Analysis and Simulation of dynamical systems: Lotka-Volterra model, FitzHugh-Nagumo model.

#### Teaching methods

Frontal lectures + lab sessions

**Recommeneded or required  
readings**

F. Verhulst. Nonlinear differential equations and dynamical systems. Springer-Verlag, Heidelberg, 2006.

R. Mattheij, J. Molenaar. Ordinary differential equations in theory and practice. SIAM, Philadelphia, 2002.

A. Quarteroni, R. Sacco, F. Saleri. Matematica Numerica. Springer 3rd ed., 2008.

M. Crouzeix, A.L. Mignot. Analyse Numeriques des Equations Differentielles. Masson, Paris 1984.

A.M. Stuart, A.R. Humphries. Dynamical Systems and Numerical Analysis. Cambridge University Press 1998.



Written/oral examination with discussion and interpretation of the models and simulations developed in the laboratory.









