UNIVERSITÀ DI PAVIA

Anno Accademico 2021/2022

NUMERICAL MODELLING

Enrollment year	2019/2020
Academic year	2021/2022
Regulations	DM270
Academic discipline	MAT/08 (NUMERICAL ANALYSIS)
Department	DEPARTMENT OF MATHEMATICS "FELICE CASORATI"
Course	MATHEMATICS
Curriculum	PERCORSO COMUNE
Year of study	3°
Period	1st semester (29/09/2021 - 14/01/2022)
ECTS	6
Lesson hours	56 lesson hours
Language	Italian
Activity type	WRITTEN AND ORAL TEST
Teacher	MOIOLA ANDREA (titolare) - 6 ECTS
Prerequisites	The skills acquired in the first classes of analysis, linear algebra and numerical analysis. Basic knowledge of Matlab or a similar language.
Learning outcomes	The course aims to integrate and extend the knowledge gained in the previous numerical analysis courses. The fundamental objective is to present several techniques of numerical modelling for the solution of boundary value problems, mostly of diffusion-advection-reaction type. We will restrict to the one-dimensional case, but the ideas and the techniques learned may be applied to much more general situations. At the end of the course, the student will be able to: study the theoretical properties of a boundary value problem, choose the most suitable numerical techniques to approximate it, analyse the stability and convergence properties of the chosen method, and implement it efficiently.

Course contents	We introduce numerical algorithms for the solution of differential boundary values problems. Elements of the MATLAB language are part of the course content. - Shooting method for linear and non-linear boundary value problems. - Diffusion-transport-reaction models. - Well-posedness of the Dirichlet problem in one dimension, maximum principle, Green function, other boundary conditions. - Numerical differentiation: finite differences; truncation error and round-off error. - Finite difference method. Existence, uniqueness and accuracy of the discrete solution. The Neumann problem. Efficient implementation. Diffusion-transport problem, upwind method. Eigenvalue problems. Non-linear problems. Uncertainty quantification. - Polynomial and trigonometric collocation method; the discrete Fourier transform and the FFT. - The weak formulation of a boundary value problem; abstract variational problems. - The Galerkin method. - The finite element method; linear and quadratic finite elements; error analysis. - Evolution problems: the heat equation, the Fourier method, the theta-method.
Teaching methods	Lectures and tutorials in the computer lab. The lectures are given at the board (either physical or virtual) and require the active participation of the students, in particular for the problem-solving parts. In the computer lab tutorials the students implement and analyse numerically the methods learned in class, alone or in little groups, with the help of the teacher. The coding assignments are distributed in advance and range from simple applications of the theory to small projects.
Reccomended or required readings	The main reference are the lecture notes prepared by the lecturer and available on the course webpage. Further references: V. Comincioli, Analisi Numerica. Metodi, Modelli, Applicazioni, McGraw-Hill, 1995. A. Iserles, A First Course in the Numerical Analysis of Differential Equations, Cambridge University Press, 2009. R.J. LeVeque, Finite Difference Methods for Ordinary and Partial Differential Equations. Steady-state and Time-dependent Problems, SIAM 2007. A. Quarteroni, R. Sacco, F. Saleri, P. Gervasio, Matematica Numerica, Springer, 2014. G. Strang, G. Fix, An Analysis of the Finite Element Method, Wellesey–Cambridge press, 2008 (1st ed. 1973).

	E. Suli, D. Mayers, An introduction to Numerical Analysis, Cambridge University Press, 2003. A. Tveito, R. Winther, Introduction to Partial Differential Equations. A Computational Approach, Springer 2005.
Assessment methods	Written and oral exam with discussion of Matlab reports. The students will have to show that they know the theoretical concepts, are able to apply them to concrete problems, can compare different strategies, can describe, implement and analyse the numerical schemes that constitute the course programme. If for health reasons it will be impossible to take the exam on site, the exam might be done in oral form.
Further information	
Sustainable development goals - Agenda 2030	$lbl_legenda_sviluppo_sostenibile