UNIVERSITÀ DI PAVIA

Anno Accademico 2019/2020

ELEMENTS OF MATHEMATICS	
Enrollment year	2019/2020
Academic year	2019/2020
Regulations	DM270
Academic discipline	MAT/08 (NUMERICAL ANALYSIS)
Department	DEPARTMENT OF CIVIL ENGINEERING AND ARCHITECTURE
Course	CIVIL ENGINEERING
Curriculum	Idraulico
Year of study	1°
Period	1st semester (30/09/2019 - 20/01/2020)
ECTS	6
Lesson hours	45 lesson hours
Language	Italian
Activity type	WRITTEN AND ORAL TEST
Teacher	MARINI LUISA DONATELLA (titolare) - 3 ECTS MOIOLA ANDREA - 3 ECTS
Prerequisites	Basic knowledge of Differential and Integral Calculus, Linear Algebra and programming.
Learning outcomes	To provide some basic instruments necessary for the theoretical and numerical analysis of Partial Differential Equations (PDE's) of interest for applications.
Course contents	The course is divided in two parts, strictly related to one another. In the first part a theoretical study of some model applicative problems described by Partial Differential Equations (PDE's) will be carried out. The second part is devoted to the numerical solution of the problems analysed in the first part of the course. In particular, some of the following arguments will be developed. GENERALITIES ON PDE's:

definition of Partial Differential Equations of order m; linear, semi-linear and quasi-linear equations.

FIRST ORDER PDE's:
Transport equation; constant and variable transport coefficient; Cauchy's problem. Solution of the homogeneous case with the method of characteristics; stability. Non homogeneous case. Hints on the case of non constant transport: rarefaction and shock wave. The case of boundary value problems.

SECOND ORDER PDE'sS:
Linear equations with constant coefficients; classification.

- Hyperbolic problems: the wave equation, D'Alembert solution. Stability and related problems. Bidimensional problems on a square: solution by separation of variables.

- Hilbert functional spaces in 1 and 2 dimensions: norms and scalar products, Cauchy-Schwarz and Poincare' inequalities.

- Elliptic problems: Poisson problem, weak (variational) formulation; equivalence with a minimum problem.

- Parabolic problems: the heat equation, uniqueness of the solution, variational formulation.

INTRODUCTION TO FINITE DIFFERENCE AND FINITE ELEMENT METHODS:
One-dimensional model problem; extension to 2 dimensions; Poisson problem. Approximation with piecewise linear finite elements; interpolation and approximation errors. 1D advection-diffusion problems: behaviour of the numerical solution in the advection-dominated case. Stabilization methods: artificial viscosity and up-wind; Hints on artificial viscosity and streamline diffusion (SUPG) schemes for two-dimensional problems.

DISCRETIZATION OF PARABOLIC PROBLEMS:
Finite Element approximation in space and theta-method in time. Hints on the case of two space dimensions.

DISCRETIZATION OF HYPERBOLIC PROBLEMS:
Semidiscretization in space with Finite Elements (continuous or discontinuous). Stabilization via artificial viscosity. Space-time Finite Element approximation. Hints on nonlinear problems.

Teaching methods	Lectures (hours/year in lecture theatre): 45 Practical class (hours/year in lecture theatre): 11 Practicals/Workshops (hours/year in lecture theatre): 0
Reccomended or required readings	S. Salsa. Partial Differential Equations in action: from Modelling to Theory, Springer Universitext, 2009. A. Quarteroni. Numerical Models of Differential Problems. Springer

	Series MS&A, Vol. 2, 2009.
Assessment methods	Written part: it consists of 2 questions on subjects developed during the ccourse. Duration: 1 hour. To pass the exam a grade of at least 18/30 must be obtained. Oral part: not compulsory, open to students who wish to increase the grade obtained in the written part. In case of failure of the oral exam the grade obtained in the written part is cancelled.
Further information	additional information can be found on my web page: http://arturo.imati.cnr.it/marini
Sustainable development goals - Agenda 2030	$lbl_legenda_sviluppo_sostenibile