



# UNIVERSITÀ DI PAVIA

Anno Accademico 2016/2017

| FINITE ELEMENTS     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
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| Enrollment year     | 2016/2017                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
| Academic year       | 2016/2017                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
| Regulations         | DM270                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |
| Academic discipline | MAT/08 (NUMERICAL ANALYSIS)                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
| Department          | DEPARTMENT OF MATHEMATICS "FELICE CASORATI"                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
| Course              | MATHEMATICS                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
| Curriculum          | PERCORSO COMUNE                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
| Year of study       | 1°                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
| Period              | 2nd semester (01/03/2017 - 09/06/2017)                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| ECTS                | 9                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| Lesson hours        | 72 lesson hours                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
| Language            | ITALIAN                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
| Activity type       | ORAL TEST                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
| Teacher             | SANGALLI GIANCARLO (titolare) - 6 ECTS<br>BOFFI DANIELE - 3 ECTS                                                                                                                                                                                                                                                                                                                                                                                                                              |
| Prerequisites       | Fundamental notions of Analysis and Numerical Analysis                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| Learning outcomes   | Numerical and theoretical study of the finite element method and its application                                                                                                                                                                                                                                                                                                                                                                                                              |
| Course contents     | <p>The aim of the course is to present the theoretical foundation of the finite element course, example of applications to the numerical solution of partial differential equations, and discuss its implementation. We will consider both diffusion (elliptic) problems and mixed problems, analyzing its stability, approximation properties. Then we will focus on mixed problems. In parallel, we will discuss and test its implementation in MATLAB language</p> <p>Extended summary</p> |

Theory lessons will cover the following topics:

- fundamentals of Functional Analysis, with a particular emphasis on the  $W^{k,p}$  spaces and on primal variational formulations of elliptic problems
- approximation theory in Sobolev spaces: Deny-Lions Lemma and Bramble-Hilbert lemma
- Lagrange interpolation on  $n$ -simplices and corresponding interpolation error for Sobolev norms
- Galerkin method for elliptic problems and error estimates: Cea Lemma and duality techniques
- Finite Element Methods for elliptic problems, with particular emphasis to the bidimensional case
- mixed formulation of elliptic problems and its Galerkin discretization: existence, uniqueness, stability of the solution, and error analysis. Some example of Finite Elements for the diffusion problem in mixed form
- elasticity problem and its FEM discretization: the volumetric locking phenomenon and some possible cures

Computer Lab lessons will address the implementation of the finite element method, in MATLAB language. In particular:

- data structure and algorithm for the triangulation of a planar region
- interpolation and numerical integration of functions on the triangulation
- local matrices and assembling
- Dirichlet and Neumann boundary condition
- finite element method for the Poisson problem in primal form with P1 elements
- implementation of the RT element
- finite element method for the Poisson problem in mixed form (Darcy problem)

REMARK: This is a tentative program. Significant changes might occur, also depending on the feedback provided by the Student during the lectures.

#### Teaching methods

Lessons and computer lab practice

#### Recommended or required readings

A. Quarteroni, A. Valli: "Numerical Approximation of Partial Differential Equations", Springer-Verlag, 1994.

Daniele Boffi, Franco Brezzi, and Michel Fortin. Mixed finite element methods and applications. Berlin: Springer, 2013.

#### Assessment methods

Oral examination

#### Further information

Oral examination

#### Sustainable development goals - Agenda 2030

[\\$lbl\\_legenda\\_sviluppo\\_sostenibile](#)