UNIVERSITÀ DI PAVIA

Anno Accademico 2020/2021

DIFFUSION AND TRANSPORT PHENOMENA	
Enrollment year	2020/2021
Academic year	2020/2021
Regulations	DM270
Academic discipline	MAT/07 (MATHEMATICAL PHYSICS)
Department	DEPARTMENT OF PHYSICS
Course	
Curriculum	Fisica della materia
Year of study	1°
Period	2nd semester (01/03/2021 - 11/06/2021)
ECTS	9
Lesson hours	72 lesson hours
Language	Italian
Activity type	WRITTEN AND ORAL TEST
Teacher	BISI FULVIO (titolare) - 6 ECTS SALVARANI FRANCESCO - 3 ECTS
Prerequisites	Basic knowledge of calculus, linear algebra, mechanics and functional analysis.
Learning outcomes	The course provides an introductory mathematical study of some peculiar time-dependent partial differential equations that describe transport and diffusion phenomena. The lectures will enlight the links between the physical properties of the systems and the mathematical properties of the corresponding models, in particular the linear Boltzmann equation and the soft matter model.
Course contents	First part (6 CFU - F. Bisi) Introduction to continuum mechanics Tensor algebra. Euclidean space; points, vectors, tensors; vector space of translations;

Cartesian triad; dyadic product; canonical basis of dyads for the vector space of tensors. Symmetric and skew tensors, axial vector of a skew tensor. Tensor invariants of first, second and third order. Orthogonal tensors, rotations. Spectral theorem; spectral decomposition.
Square-root lemma, polar decomposition theorem.

Tensor calculus.
Differentiability of a linear application. Theorem of the regularity of the inverse linear application. Derivative of a product, of a composite function. Gradient (scalar / vector); divergence (vector / tensor). Curl of a vector. Normalized curves, circulation. Divergence theorems for scalar, vector or tensor fields; localization theorem; Stokes' theorem.

Continuum model.
Deformation, deformation gradient; decompositions. Rigid deformations, translations, rotations; small deformations, infinitesimal rigid displacements. Motion; material description and spatial description (Lagrangian and Eulerian points of view). Material and spatial temporal derivatives. Velocity ??gradient. Theorems transport of vorticity (spin). Transport theorems (Reynolds' volume theorem). Spin, theorem of transport of circulation and of vorticity. Density in motion. Mass conservation, continuity equations Momentum and angular momentum. Balance equations. Cauchy fundamental theorem for the stress tensor. Thermodynamic quantities and constitutive equations. Classic materials: perfect, incompressible, and barotropic fluids; perfect fluids and Euler equations; Newtonian fluids and Navier Stokes equations, diffusion of vorticity. Uniqueness and stability for solutions of a viscous flow problem.

Heat equation as a paradigm of diffusion. Boundary conditions: Dirichlet, Neumann, Robin, mixed. Uniqueness of the solution by energy methods. Weak and strong maximum (minimum) principle; corollaries. Parabolic riscaling. Fundamental solution. Usage of the fundamental solution in a homogeneous or an inhomogeneous Cauchy problem.

PME: standard porous medium equation (non-linear heat equation). Finite speed of propagation: steady solutions, separable-variable solutions, wave solutions, Barenblatt fundamental solutions. Incompressible fluid in a porous medium. S

Second Part (3 CFU - F. Salvarani)
Transport and diffusion

Introduction.
Origin of transport and diffusion equations: the random walk, heat equation and free transport equation. The formalism of kinetic theory. Transport and diffusion scalings. Formal relaxation from transport to distribution.

The linear free transport equation.
The Cauchy problem. The method of characteristics, estimates. The initial-boundary value problem. Incoming, outgoing and characteristic boundary. Backwards exit time, regularity. Source and absorption terms. The maximum principle. The stationary transport equation: existence

and uniqueness theorem, maximum principle. Renewal boundary conditions.

Introduction to finite difference numerical methods for the free transport equation.
Consistency, stability and convergence for finite differences numerical methods. The Lax-Friedrichs, Upwind and Diamond schemes. Their fundamental properties.

The linear Boltzmann equation.
The Cauchy problem: existence and uniqueness, estimates and positivity of the solution. The initial-boundary value problem for the linear Boltzmann equation: influx conditions, specular and diffuse reflection. The Darrozes-Guiraud lemma. Existence and uniqueness of the solution.

The diffusion limit for the linear Boltzmann equation.
The heat equation in a bouded domain, basic properties. Diffusive scaling and Hilbert development. Convergence theorem.

Critical calculus.
Asymptotic behavior in time for the linear Boltzmann equation.

The heat equation.
The Cauchy problem for the heat equation: existence and uniqueness of the solution, qualitative properties (maximum principle, regularizing effect, irreversibility, infinite propagation speed). Asymptotic behavior in time.

Homogenization.
Applications of the two-scale homogenization theory to transport and diffusion.

The porous media equation.
Self-similar solutions. Basic properties. The Cauchy problem.

Systems of diffusion equations.
Introduction to cross-diffusion equations. The Maxwell-Stefan system. Turing instability.

Teaching methods

Lectures (24+48 hours in lecture theatre)

Reccomended or required readings

M. E. Gurtin. An Introduction to Continuum Mechanics. Academic Press (NY), 1981.

S. Salsa: Partial Differential Equations in Action: From Modelling to Theory", Springer (Milan), 2009.

G. Allaire, X. Blanc, B. Despres, F. Golse. Transport et diffusion. Les éditions de l'Ecole Polytechnique. Palaiseau 2019

L. C. Evans. Partial differential equations. American Mathematical Society. Providence 1998

	J. L. Vázquez. The Porous Medium Equation: Mathematical Theory. Clarendon Press. Oxford 2007
Assessment methods	Oral assessment.
Further information	
Sustainable development goals - Agenda 2030	$lbl_legenda_sviluppo_sostenibile