



### FUNCTIONAL ANALYSIS

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| <b>Enrollment year</b>     | 2018/2019  |
| <b>Academic year</b>       | 2018/2019  |
| <b>Regulations</b>         | DM270  |
| <b>Academic discipline</b> | MAT/05 (MATHEMATICAL ANALYSIS)   |
| <b>Department</b>          | DEPARTMENT OF PHYSICS  |
| <b>Course</b>              |  |
| <b>Curriculum</b>          | Fisica teorica   |
| <b>Year of study</b>       | 1°   |
| <b>Period</b>              | 1st semester (01/10/2018 - 18/01/2019)   |
| <b>ECTS</b>                | 9  |
| <b>Lesson hours</b>        | 78 lesson hours  |
| <b>Language</b>            | Italian  |
| <b>Activity type</b>       | WRITTEN AND ORAL TEST  |
| <b>Teacher</b>             | MORA MARIA GIOVANNA (titolare) - 9 ECTS  |
| <b>Prerequisites</b>       | Multivariable differential and integral calculus. Lebesgue measure and integration. Basic notions of linear algebra and topology.  |
| <b>Learning outcomes</b>   | At the end of the course students will know the main results and principles of abstract Functional Analysis. Through the exercise sessions students will learn how to apply the theoretical results to explicit problems. Moreover, they will be able to work autonomously on the formulation and the analysis of problems of Mathematical Analysis in spaces of infinite dimension. |
| <b>Course contents</b>     | <p>Norms and scalar products. Normed spaces. Bounded linear operators. Topological dual space.</p> <p>Banach spaces. Hahn-Banach Theorem: analytical and geometrical forms and their consequences. Baire Lemma. Banach-Steinhaus Theorem. Open Mapping Theorem, Closed Graph Theorem, and their</p>  |

consequences.

Weak\* topology, weak topology, and their properties. Banach-Alaoglu Theorem. Reflexive spaces. Separable spaces.

$L^p$  spaces. Reflexivity and separability of  $L^p$ . Riesz Representation Theorem. Approximation by convolution. Ascoli-Arzelà Theorem. Fréchet-Kolmogorov Theorem.

Hilbert spaces. Projection on a convex closed set. Riesz Representation Theorem for the dual space. Lax-Milgram Theorem. Complete orthonormal systems.

Compact operators. Adjoint of a bounded operator. The Fredholm Alternative. Spectrum of a compact operator. Spectral decomposition of a compact self-adjoint operator. Integral operators. Application to Sturm-Liouville problems.

#### Teaching methods

Lectures and exercise sessions. Exercises will be assigned to students a few days in advance, before being discussed in the exercise session. Lecture notes will be provided on the KIRO webpage.

#### Reccomended or required readings

H. Brézis: Functional analysis, Sobolev spaces and partial differential equations. Springer, 2011.

W. Rudin: Real and complex Analysis. McGraw-Hill, 1987.

#### Assessment methods

The exam consists into a written test and an oral exam. In the written test students will be asked to solve some exercises and to present the statement and proof of a theorem. Students will be admitted to the oral exam only if they obtain a score of at least 15/30 in their written test. The oral exam will concern results, proofs, examples discussed in the course.

#### Further information

The exam consists into a written test and an oral exam. In the written test students will be asked to solve some exercises and to present the statement and proof of a theorem. Students will be admitted to the oral exam only if they obtain a score of at least 15/30 in their written test. The oral exam will concern results, proofs, examples discussed in the course.

#### Sustainable development goals - Agenda 2030

[\\$|bl legenda sviluppo sostenibile](#)