UNIVERSITÀ DI PAVIA

Anno Accademico 2014/2015

FUNCTIONAL ANALYSIS	
Enrollment year	2014/2015
Academic year	2014/2015
Regulations	DM270
Academic discipline	MAT/05 (MATHEMATICAL ANALYSIS)
Department	DEPARTMENT OF PHYSICS
Course	
Curriculum	FISICA TEORICA
Year of study	1°
Period	1st semester (13/10/2014 - 23/01/2015)
ECTS	9
Lesson hours	78 lesson hours
Language	ITALIAN
Activity type	ORAL TEST
Teacher	SCHIMPERNA GIULIO FERNANDO (titolare) - 9 ECTS
Prerequisites	Differential and integral calculus for functions of one or more variables. Lebesgue theory of measure and integration. Basic notions of linear algebra.
Learning outcomes	The course is aimed at: a) presenting the basic notions of the theory of Hilbert and Banach spaces with particular emphasis on the latter; b) showing how the techniques of Functional Analysis may be applied to solving concrete mathematical problems; c) illustrating the interplay between theory, results and applications.
Course contents	1) norms, normed spaces, Banach and Hilbert spaces, duality; 2) Hahn-Banach theorem and applications; 3) Banach-Steinhaus theorem and its consequences; unbounded linear operators; 4) weak topologies, reflexivity and separability;

5) Lp spaces;
6) Hilbert spaces;
7) Sobolev spaces of functions of one scalar variable.

Extended summary

1. Norms and scalar products. Topological vector spaces. Completeness. Banach and Hilbert spaces. Some examples (spaces of continuous / integrable functions). Duality. Dual spaces. Bounded linear operators.

2. Analytical form of the Hahn-Banach theorem. Applications of the theorem. Duality mapping. Geometrical form of the Hahn-Banach theorem. Convex functions;
convex conjugate function; subdifferential; Fenchel-Moreau theorem.

3. Some fundamental results of the Banach space theory: theorems of Banach-Steinhaus, of the open mapping, and of the closed graph. Consequencees. Unbounded linear operators. Closed operators. Orthogonality relations.

4. Reflexivity. Important examples of reflexive spaces. Seminorms; Minkowski functionals, locally convex spaces, Frechet spaces. Weak and weak* topologies. Weak compactness theorems.
Separability.

5. Lp spaces. Fundamental inequalities. Riesz representation theorems. Reflexivity and separability of Lp. Convolutions. Mollifiers. Ascoli's theorem. Strong compactness in Lp.

6. Hilbert spaces. Projections on a closed convex subset. Stampacchia and Lax-Milgram theorems. Hilbert bases and sums.

7. Sobolev spaces in space dimension 1. Regularity of Sobolev functions. Reflexivity and separability of Sobolev spaces. Extension theorems. Sobolev embeddings. Traces. Applications to partial differential equations.

Teaching methods	Lessons, partly devoted to the resolution of exercises
Reccomended or required readings	- Brezis, Analisi Funzionale, Liguori Editore (an English edition, published by Springer, is also available) - Lecture notes by Gianni Gilardi
Assessment methods	Written and oral exam
Further information	The written test is optional and will be proposed only once per year, just after the end of the course
Sustainable development goals - Agenda 2030	$lbl_legenda_sviluppo_sostenibile